Section 3.4

Slope and Rate of Change
Learning objectives

- Find the slope of a line, given two points on the line
- Find the slope of a line given its equation, including horizontal and vertical lines
- Compare the slopes of parallel and perpendicular lines
- Slope as a rate of change
- Vocabulary: slope, rise, run, zero slope, undefined slope, parallel, perpendicular
Finding the slope, given two points

- The steepness of a line is called its **slope**.
- The slope is the amount of vertical change, or **rise**, divided by its horizontal change, or **run**.
- We take the difference of the ‘y’ values for the rise, of vertical change. $6 - 2 = 4$
- The difference of ‘x’ values is the run. $4 - 1 = 3$
- So the slope is the rise-over-run, or $4/3$
It makes no difference what two points on a line are chosen to find its slope.

- Use points (1,2) and (4,6)
  - Rise = 6 – 2 = 4
  - Run = 4 – 1 = 3
  - Slope = 4/3

- Use points (4,6) and (7,10)
  - Rise = 10 – 6 = 4
  - Run = 7 – 4 = 3
  - Slope = 4/3

- Use points (1,2) and (7,10)
  - Rise = 10 – 2 = 8
  - Run = 7 – 1 = 6
  - Slope = 8/6 = 4/3

Slope = rise over run
Slope is denoted by the letter $m$

- Given two points $(x_1, y_1)$ and $(x_2, y_2)$:
  - the **rise** is the difference between $y$-coordinates
  - the **run** is the difference between $x$-coordinates
- The **slope** $m$ is the rise divided by the run:
  $$m = \frac{y_2 - y_1}{x_2 - x_1}$$
Find the slope of the line between (-1,5) and (2,-3). Graph the line

- Let \((x_1, y_1) = (-1,5)\) and \((x_2, y_2) = (2,-3)\)
- So \(m = \frac{y_2 - y_1}{x_2 - x_1}\)
- \(m = \frac{-3 - 5}{2 - (-1)}\)
- \(m = -\frac{8}{3}\)
- This is a line that moves 8 spaces \textit{down} vertically for every 3 spaces \textit{right} it goes.
- A negative slope goes down (negative \(y\)) when it also goes right (positive \(x\))
Find the slope of the line through (-1,-2) and (2,4). Graph the line.

- \((x_1,y_1) = (-1,-2)\)
- \((x_2,y_2) = (2,4)\)
- So \(m = \frac{y_2 - y_1}{x_2 - x_1}\)
- \(m = \frac{4 - (-2)}{2 - (-1)}\)
- \(m = \frac{6}{3} = 2\)
- If \(m\) is positive, then the line goes up as it goes right.
Find the slope of the line that passes through:

(1, 7) and (6, 5)
Finding the slope of a line, given its equation

- A slope on a line is defined by two points on the line.
- We have taken an equation with two variables, and found points that are in the solution of that equation.
- So to find the slope of a line, given its equation, we find two of its points, and call them \((x_1, y_1)\) and \((x_2, y_2)\).
- Then we apply the slope formula \(m = (y_2 - y_1)/(x_2 - x_1)\).
Find the slope of the equation $y = 3x - 2$

- If $x$ is set to 0, then
  - $y = 3(0) - 2; y = -2$

- If $x$ is set to 1, then
  - $y = 3(1) - 2; y = 1$

- 2 pts are (0, -2) and (1, 1)

- Then $m = (1 - (-2))/(1-0)$
- $m = 3/1 = 3$

- What else do we notice about the number ‘3’?
The significance of the number 3

- We calculated that the slope of the equation $y = 3x - 2$ was $m = 3$
- The slope of the line is the same as the coefficient of the $x$-term “3x”
- In equations of the form: $y = mx + b$
- The coefficient of the ‘x-term’ is the slope ($m$) of the line

This is on the top of page 218
Problem is, not all equations are presented in $y = mx + b$ format

- Find the slope for the line $-2x + 3y = 11$
- We have to get equation in the $y = mx + b$ format
- First, add $2x$ to each side
  - $3y = 2x + 11$
- Divide both sides by 3
  - $y = \frac{2}{3}x + \frac{11}{3}$
- The coefficient of the $x$-term is $\frac{2}{3}$
- So the slope $m = \frac{2}{3}$
Find the slope for each equation

- $-y = 5x - 2$

- $x + y = 12$

- $9x + y = -12$

- $2x - 7 = 0$
Finding slopes of horizontal and vertical lines

- What is the slope of the line: \( y = -1 \)?

- No matter what \( x \) is, \( y \) is always \(-1\). There is no “rise”, only “run”

- So: \( m = \text{rise}/\text{run} = 0 \)

- A horizontal line has slope of 0.
What is the slope of a vertical line?

- Find the slope of the line: \( x = 4 \)
- No matter what \( y \) is, \( x \) will always be 4. This is a vertical line.
- Recall that \( m = \text{rise/run} \)
- There is no run, only rise
- Since run = 0, and we can’t divide things by 0, the slope \( m \) of a vertical line is “undefined”
The red line has positive slope $m > 0$
The blue line has negative slope $m < 0$
The red line has zero slope: $m = 0$
The blue line has undefined slope
Slopes of parallel and perpendicular lines

- Two lines are parallel if they can never intersect.
- Parallel lines have the same slope.
- Here are the graphs of two equations:
  \[ y = -2x + 4 \]
  \[ y = -2x - 3 \]
- They have the same slope (-2) so they have the same slope.

\[ Y = -2x + 4 \]
\[ Y = -2x - 3 \]
Parallel lines (pg. 220)

- Nonvertical parallel lines have the same slope and different $y$-intercepts.
- They will cross the $y$-axis in different places.
- The space between the two will always remain the same.
Perpendicular lines

- Two lines are perpendicular if they meet at a 90-degree (right) angle.
- Two lines have slopes $m_1$ and $m_2$
- The product of $m_1$ & $m_2$ is always -1
- $m_1 \times m_2 = -1$
Perpendicular lines (pg. 220)

- Two nonvertical lines are perpendicular if the slope of one is the negative reciprocal of the slope of the other.
  \[ m_1 \times m_2 = -1 \]

- The negative reciprocal of 4 is \(-\frac{1}{4}\)
- The negative reciprocal of \(-\frac{6}{5}\) is \(\frac{5}{6}\)
Graphs of $y = 4x + 1$ & $y = -1/4x - 3$

- The slope of $y = 4x + 1$ is $m_1 = 4$
- The slope of $y = -1/4x - 3$ is $m_2 = -1/4$
- Since $m_1 * m_2 = 4 * -1/4 = -1$, these two lines are perpendicular
Are these pair of lines parallel, perpendicular, or neither?

- \( y = -1/5x + 1 \) \quad \text{and} \quad 2x + 10y = 3

- In the first line, it is in \( y = mx + b \) format already, and we see that \( m = -1/5 \)
- The second line needs to be solved for \( y \):
  - First: subtract 2\( x \) from both sides, then divide by 10
  - \( 10y = -2x + 3 \); \quad y = -1/5x + 3/10
- The second line also has slope \( m = -1/5 \)
- Therefore these two lines are: parallel
Are these two lines parallel, perpendicular, or neither?

- $x + y = 3$; $-x + y = 4$
- Both equations must be solved for $y$:
  - $x + y = 3$; subtract $x$ from both sides; $y = -x + 3$
  - In this equation, $m = -1$
  - $-x + y = 4$; add $x$ to both sides; $y = x + 4$
  - In this equation, $m = 1$
- The two slopes $-1 \times 1 = -1$; the product of the two slopes is $-1$
- Therefore these two lines are: perpendicular
Parallel or perpendicular or neither?

- $3x + y = 5; \ 2x + 3y = 6$
- $3x = 2y + 3; \ 2x + 3y = 2$
Find the slope of a line that is a) parallel and b) perpendicular to the lines below

- The line going through (-5,-5) and (-1,-1)
  - Rise: \(-1 - (-5) = -1 + 5 = 4\)
  - Run: \(-1 - (-5) = -1 + 5 = 4\)
  - \(m = \text{rise/run} = 4/4 = 1\)
  - Slope of a parallel line = 1
  - Slope of a perpendicular line = -1, since \(-1 \times 1 = -1\)

- The line going through (-2,10) and (5,-4)
Slope as a rate of change

- If a house roof is described as a 30% pitch, that means the slope is 30% or 0.3
- The rise/run is 0.3, the roof rises 3 feet vertical for every 10 horizontal feet

- How about a highway with 10% grade?
- The slope is 10% or 0.1
- The road rises 1 foot for every 10 horizontal feet
- That is a very steep road. Most Illinois roads never even approach 1% grade
Slope problems

- An incline ramp in a warehouse rises 16 inches for each horizontal distance of 17 feet. What is the slope as a grade (by percent)?
One question quiz

- No need to sign this quiz
- What is the slope of a line with points (3,2) and (5,8)?
- Remember slope equals rise over run
- You can leave when it is done