Section 3.2

Graphing linear equations
Learning objectives

- Graph a linear equation by finding and plotting ordered pair solutions
- Graph a linear equation and use the equation to make predictions
- Vocabulary: *linear equation in two variables; graph of the equation; horizontal line, vertical line*
Linear equation in two variables

- We know that equations with 2 variables have more than one solution, and each solution is an “ordered pair” of numbers.

- Here are some solutions of $x + y = 4$. If $x$ is 2 and $y$ is 2, that is one solution and it is also ordered pair (2,2)

- So is (0,4), (4,0), (-2,6) and (6,-2)

- Can you see the line that is starting to form? What about all the spaces inbetween?
If we came up with *all* the points on the line including fractions, it would resemble a line.

In fact, every point on this line is a solution to \( x + y = 4 \). This line is the *graph of the equation* \( x + y = 4 \).

A line has an arrow on both ends which means the line goes on forever.
The equation \( x + y = 4 \) is called a linear equation in two variables.

A linear equation in two variables is written in the form:

\[ Ax + By = C \]

Where \( A, B, \) and \( C \) are real numbers and \( A \& B \) are not 0.

This is also called “standard form”.

The graph of a linear equation in two variables is always a straight line.

Linear equations in two variables can also appear as:

\[ -2x = 7y \quad y = \frac{1}{4} x + 2 \quad y = 7 \]

Why? All of these can be simplified to: \( Ax + By = C \) using addition property and multiplication property of equality.
Graphing Linear Equations

- To graph a linear equation, we usually find 3 of its solutions (ordered pairs)
- Then we plot the three points on the rectangular coordinate plane, then draw a straight line through the points
- For $2x + y = 5$, we can find solutions $(1,3)$, $(0,5)$, and $(3,-1)$, then fill in a table as seen to the right

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>
Where did we get (1,3), (0,5) and (3,-1)?

- First, replace $x$ with 1 and solve for $y$:
  - $2(1) + y = 5$; subtract 2 from each side
  - $2 - 2 + y = 5 - 2$; $y = 3$, and ordered pair (1,3) is a solution.

- Replace $x$ with 0 and solve for $y$:
  - $2(0) + y = 5$;
  - $y = 5$; and ordered pair (0,5) is a solution

- Replace $x$ with 3 and solve for $y$:
  - $2(3) + y = 5$; subtract 6 from both sides
  - $y = -1$; and ordered pair (3,-1) is a solution
Then plot the points on the graph

<table>
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</tr>
<tr>
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<td>-1</td>
</tr>
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</table>
For each equation, find three ordered pair solutions by completing the table

1. \( x - y = 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

2. \( y = -\frac{1}{3}x - 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>-4</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

3. \( y = -3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
When graphing a linear equation in two variables, if it is:

- Solved for $y$, it may be easier to find ordered-pair solutions by choosing $x$ values.
  
  For example: $y = \frac{1}{4}x + 8$. Choose three $x$ values, and find the $y$ values.

- Solved for $x$, it may be easier to find ordered pair solutions by choosing $y$ values.
  
  For example: $x = \frac{2}{3}y$. Choose three $y$ values, and find the $x$ values.
Graph \(-5x + 3y = 15\)

- Let \(x = 0\). \(3y = 15, y = 5\)

- Let \(y = 0\). \(-5x = 15, x = -3\)

- Let \(x = -2\).
  - \(-5(-2) + 3y = 15\)
  - \(10 + 3y = 15\)
  - Subtract 10 from both sides
  - \(3y = 5\)
  - \(y = 5/3\) or \(1 \frac{2}{3}\)

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>1 \frac{2}{3}</td>
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Graph \(-5x + 3y = 15\)

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</tr>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>1 (\frac{2}{3})</td>
</tr>
</tbody>
</table>
Graph the following linear equations. Draw your table, then chart your graph.

- $x + y = 0$
- $y = -2x - 1$
- $x - 2 = 0$
- $y = -2$
Using linear equations to model real data

- The number of people $y$ (in thousands) expected to be employed in the US as medical assistants is estimated by the equation $y = 31.8x + 180$, where $x$ is the number of years after 1995. Graph the equation to find the number of medical assistants employed now (2010).

- The year 1995, therefore, is equal to $x = 0$
- The year 1997 is equal to $x = 2$
- The year 2002 is equal to $x = 7$

- To graph the equation, find $y$ (the number of assistants in thousands) at $x = 0, 2, & 7$ and plot the graph.
Estimating the number of medical assistants (cont.)

- \( y = 31.8x + 180 \)
- \( x = 0, y = 180 \)

- \( x = 2, y = 31.8(2) + 180 \)
  - \( y = 63.6 + 180 = 243.6 \)

- \( x = 7, y = 31.8(7) + 180 \)
  - \( y = 222.6 + 180 = 402.6 \)

<table>
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<tbody>
<tr>
<td>0</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>243.6</td>
</tr>
<tr>
<td>7</td>
<td>402.6</td>
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Estimating the number of Medical assistants (cont.)

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Draw the line out to x=15 to determine there will be ~650 medical assistants in 2010.
The value of a house \((y)\) increases in value \(x\) years by the formula \(y = 7500x + 120000\)

- Fill in the table, graph the equation, and write the sentence describing the value of a house in year 5

<table>
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<tr>
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</tbody>
</table>

- \(x = 0, y =\)
- \(x = 2, y =\)
- \(x = 5, y =\)
Homework for section 3.2 and 3.3

- It’s going to need to have 3 parts:
  - Your equation work for 3 points
  - Your x, y table with three rows
  - Your graph on the rectangular coordinate plane
- Recall you were given a copy of the rectangular coordinate plane worksheet on the first night of class. Make lots of copies for your homework. Label each graph with the number of the problem it represents.
- I will try to have extra copies at class for you
Section 3.3

Intercepts
Learning objectives

Identify intercepts of a graph
Graph a linear equation by finding and plotting intercept points
Identify and graph vertical and horizontal lines

Vocabulary: x-intercept, y-intercept, vertical line, horizontal line
Identifying intercepts

- The graph of \( y = 4x - 8 \) is shown. This graph crosses the \( y \)-axis (vertical) at the point (0, -8). This point is the \textit{y-intercept}.

- In a \( y \)-intercept, the first number of the point (the \( x \)) is always 0.

- The graph crosses the \( x \)-axis (horizontal) at (2, 0). This point is the \textit{x-intercept}.

- The second number of the \( x \)-intercept (the \( y \)) is always 0.
X and y intercepts

- If a graph crosses the x-axis at (2,0) and the y-axis at (0,-8), then (2,0) and (0,-8) are its x-intercept and y-intercept, respectively.
- Note that for all x-intercepts, the y-value is always 0, and for all y-intercepts, the x-value is always 0.
- Sometimes you will see the y-intercept simply written as -8 – that implies the y-intercept is really (0,-8)
- If you see the x-intercept written simply as 2, that means the x-intercept is really (2,0)
- For a graph that goes through the point (0,0), the x-intercept and y-intercept is 0
Identify the x and y intercepts

- x-intercept: (-3,0)
- y-intercept: (0,2)

- x-intercepts: (-4,0), (-1,0)
- y-intercept: (0,1)
Identify the intercepts
Identify the intercepts
Finding and plotting intercepts

- Finding intercepts should be easy, since one point is always 0.
- Remember, the x-intercept is the point where the graph crosses the horizontal axis – which is where \( y = 0 \)
- The y-intercept is where \( x = 0 \)
- To find the y-intercept, set \( x = 0 \) and solve for \( y \)
- To find the x-intercept, set \( y = 0 \) and solve for \( x \)
Graph \( x - 3y = 6 \) by finding and plotting its intercepts

- First, let \( y = 0 \)
  \[ x - 3(0) = 6; \ x = 6 \]

- Let \( x = 0 \)
  \[ (0) - 3y = 6; \ y = -2 \]

- Plot a third point: \( x = 3 \)
  \[ (3) - 3y = 6; \ subtract \ 3 \ from \ both \ sides \]
  \[ -3y = 3; \ y = -1 \]

- Now draw your graph

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
6 & 0 \\
0 & -2 \\
3 & -1 \\
\hline
\end{array}
\]
Graph $x = -2y$ by finding and plotting its intercepts

- Let $y = 0$ be the $x$-intercept; $x = -2(0) = 0$
- Let $y = 1$; $x = -2(1) = -2$
- Let $y = -1$; $x = -2(-1) = 2$

The ordered pairs are $(0,0)$, $(-2,1)$ and $(2,-1)$.  

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

Graph each linear equation by finding and plotting its intercepts

- $x - y = 2$

- $x - y = -3$
Graph each linear equation by finding and plotting its intercepts

- \(2x + 4y = 8\)
- \(x - 3y = 0\)
Graph each linear equation by finding and plotting its intercepts

- \( y = 3x + 3 \)
- \( y = -2x - 4 \)
Graphing vertical and horizontal lines

- Graph $x = 2$
- This is the same as the equation $x + 0y = 2$
- So, basically, $y$ can be anything, and $x$ is always 2
- Any ordered pair where the first number is 2 will satisfy the equation $x = 2$
- The graph is a *vertical line* with $x = 2$

This line has NO $y$-intercept because $x$ is never 0

The graph of $x = c$, where $c$ is a real number, is a vertical line with $x$-intercept $(c,0)$
Graphing horizontal lines

- Graph: $y = -2$
- This is the same as the equation $0x + y = -2$
- For any $x$ value, $y = -2$. There is no $x$-intercept.
- The graph is a horizontal line with $y$-intercept $-2$
- The graph of $y = c$, where $c$ is a real number, is a horizontal line with $y$-intercept $(0,c)$
Identify the type of equation (horizontal or vertical line)

- $x = -3$
- $y = 2$
- $x + 3 = 5$