Section 2.5

Formulas and problem solving
Learning objectives:

- Given a formula and values, solve for its unknowns
- Solve a formula or equations for one of its variables
- Solve word problems
- Vocabulary: formula, perimeter, area, volume
Using formulas to solve problems

- A formula describes a known relationship among quantities.
- For example, the formula \( d = r \times t \) describes the relationship of distance as a product of rate and time.
- If we know the distance we drove was 100 miles and it took us 2 hours to get there, we can use the formula to solve for \( r \), the average rate we drove.
- Substitute 100 for \( d \) and 2 for \( t \) gives you:
  - \( 100 = r \times 2 \); divide both sides by 2
  - \( 50 = r \). The average speed we drove was 50 miles per hour.
Example 1: Alaska glacier

- Portage glacier in Alaska is 31,680 feet long, and it moves 400 feet a year. How long does it take for the whole glacier to move past a certain point?
- Remember $d = r \times t$
- What are we solving for, $d$, $r$, or $t$? “How long” = time = $t$
- What is $d$? $d$ is 31,680 feet
- What is $r$? $r$ is 400 feet/yr
- So 31,680 = 400 $\times$ $t$. Divide both sides by 400
- Therefore 79.2 = $t$. It will take the glacier over 79 years to completely pass by a specific spot.
Example 2 - Garden

- We can afford to buy 140 feet of fence for a rectangular garden. If the garden is to be 30 feet wide, what is its length?

- UNDERSTAND: the distance of the boundary around the garden, or any area, is called its perimeter.

- The formula for perimeter is \( t = 2w + 2l \)

- What is the total perimeter (\( t \)) we can fence?

- What is the width (\( w \))?

- What is the length? Solve the formula for the length (\( l \)):
  \[ 140 = 2l + 2(30) \]
Example 3: Fahrenheit to Celsius

- If the temperature is 59 degrees F, what is it in C?
- The formula for converting degrees Fahrenheit to degrees Celsius is: \( F = \frac{9}{5} C + 32 \)
- What is \( F \)?
- Solve for \( C \):
Substitute given values and solve for the unknown variable.

- Distance formula: $d = rt; \ t = 9; \ d = 63$

- Perimeter of a rectangle: $P = 2l + 2w; \ P = 32; \ w = 7$
Substitute given values and solve for the unknown variable.

- Volume of a pyramid: \( V = \frac{1}{3}bh; \ V = 40; \ h = 8 \)

- Simple interest: \( I = prt; \ I = 23; \ p = 230; \ r = 0.02 \)
Solving a Formula for a Variable

- In math, it is said that the formula $d = rt$ has been “solved for $d$” because $d$ is by itself on one side of the equal sign.
- To solve for $t$, we would divide both sides by $r$, treating $d$ and $r$ as if they were real numbers, not variables.
- After dividing both sides by $r$, $t = d/r$. 
Solving Equations for a Specified Variable (page 134)

- Multiply on both sides to clear the equation of fractions
- Use the distributive property to remove parenthesis
- Simplify each side of the equation by combining like terms
- Isolate the specified variable on its own side
- Divide on both sides to simplify the variable
Example 5: Volume of a box

- Solve $V = l*w*h$ for $l$
- To solve for $l$, divide both sides by $w*h$
- Therefore, $l = V/wh$
- Remember, it does not matter which side of the equal side you choose to solve the variable on
Example 6: Slope of a line

- The equation for a line is \( y = mx + b \)
- Solve the equation for \( x \)
- First, subtract \( b \) from both sides: \( y - b = mx + b - b \)
- This results in: \( y - b = mx \)
- Then divide both sides by \( m \)
- The result is \( (y - b)/m = x \)
Example 7: Solve $P = 2l + 2w$ for $w$

- Recall that this is the perimeter formula
- Subtract $2l$ from both sides
  \[ P - 2l = 2w \]
- Then divide both sides by 2
  \[ (P - 2l)/2 = 2w/2 = w \]
Solve each formula for the specific variable

- Area of triangle: $A = \frac{1}{2} bh$ : solve for $b$

- Perimeter of a triangle: $P = s_1 + s_2 + s_3$ ; solve for $s_3$
Solve each problem:

- Convert 102 degrees F to C. \((F = \frac{9}{5}C + 32)\)

- You have 70 meters of fence. The length of a rectangular area is 1 meter less than twice the width. What is the length and width?
Solve this one, too

- Chris and Alicia drove 476 miles. They left their house at 7am and arrived at 4pm. They stopped for an hour for lunch. What was their average rate of speed?